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A PARAMETER ADJUSTMENT LAW FOR MULTIVARIABLE PID CONTROL SYSTEMS WITH DISTURBANCE ATTENUATION PERFORMANCE

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ABSTRACT: This paper shows a new parameter adjustment law of PID controllers for MIMO linear system with guaranteed disturbance attenuation performance. In the proposed design approach, the design problem of PID parameters is reduced to the problem of static output feedback controllers for MIMO linear systems. In this paper, we show that sufficient conditions for the existence of the proposed PID control system can be reduced to solvability of linear matrix inequalities (LMIs). Finally, a simple numerical example is shown to effectiveness of the proposed PID control system.

Key words: PID control, Parameter adjustment laws, MIMO systems, Matrix inequalities

INTRODUCTION

It is well known that the PID control strategy has been widely used in various industrial/practical control systems, and it is one of the most famous classical feedback control strategies. In most of PID controller design problems, single-input/single-output models as the controlled system are considered. However, in practical applications, controlled systems are modeled as coupled multiple-input multiple-output (MIMO) high-order systems. Therefore, it is especially important to develop effective design methods for multivariable PID control systems for such MIMO high-order systems, and a lot of researchers have been proposed various design methods for multivariable PID controllers. There are two approaches for designing multivariable PID controllers; One approach is "frequency domain approach" which is based on generalized Nyquist stability theorem. However, the frequency domain approach is not convenient to work due to their complexity. The other one for designing multivariable PID control systems is "time-domain approach", some design methods have been presented. In the existing results for the time-domain approach, the PID controller design problem is transformed into the static output feedback control one, and sufficient conditions for the existence of the multivariable PID controller are described as linear matrix inequalities (LMIs). Additionally, there are some results for multivariable PID control systems with online adjustment laws for PID parameters. However, the existing result needs to satisfy ASPR (Almost Strictly Positive Real) characteristics, and thus the design procedure is complex.

This paper proposes a new online-adjustment law for PID parameters which achieves disturbance attenuation performance for MIMO linear systems. In the proposed controller design approach, although the PID controller design problem is transformed into the static output feedback control one, ASPR characteristics does not introduced. We show that sufficient conditions for the existence of the proposed PID controllers can be reduced to solvability of LMIs in this paper. Finally, to show the effectiveness of the proposed PID controller, a simple numerical example is included.

This paper is organized as follows. Firstly, we describe the controlled system considered in this paper. Next, we show our main results. Finally, a simulation is shown to illustrate the effectiveness of the proposed control system. Here we show numerical symbols used in this paper. Throughout this paper, for a matrix A, $A < 0$ (resp. $A > 0$) means that A is negative definite (resp. positive-definite). A^T and A^{-1} show its transpose and its inverse, respectively. Moreover, $Tr\{A\}$ and diag(A_1, A_2, ..., A_n) are trace of A and the block diagonal matrix composed of matrices A_1, A_2, ..., A_n. I_n is n-th order identity matrix, for a vector a, $\|a\|$ denotes the standard Euclidian norm and for a matrix A, $\|A\|$ represents a its induced norm.

PROBLEM FORMULATION

Consider the MIMO linear system described by the following state equation:

$$\begin{gathered}\frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{x}(\boldsymbol{t}) = \boldsymbol{Ax}(\boldsymbol{t}) + \boldsymbol{Bu}(\boldsymbol{t}) + \boldsymbol{\Gamma}\boldsymbol{\omega}(\mathbf{t}),\\ \boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{Cx}(\boldsymbol{t}),\end{gathered} \tag{1}$$

where $x(t) \in \boldsymbol{R}^n, u(t) \in \boldsymbol{R}^m$, $y(t) \in \boldsymbol{R}^l$ are the state, the control input, and the measurement output, respectively. Moreover $\omega(t) \in \boldsymbol{R}^p$ is the disturbance input, and the matrices $A \in \boldsymbol{R}^{n\times n}$, $B \in \boldsymbol{R}^{n\times p}$, $\Gamma \in \boldsymbol{R}^{n\times p}$, and $C \in \boldsymbol{R}^{l\times n}$ represent the system parameters. For the MIMO linear system of (1), we define the following PID control law:

$$u(t) = K_p y(t) + K_i \int_0^t y(t)dt + K_d \frac{d}{dt} y(t). \tag{2}$$

In (2), $K_p \in \boldsymbol{R}^{m\times l}$, $K_i \in \boldsymbol{R}^{m\times l}$, $K_d \in \boldsymbol{R}^{m\times l}$ are the PID parameters. Note that use the derivative of the measurement output $\boldsymbol{y}(\boldsymbol{t})$ is not desirable. Therefore, we introduce the following approximate derivative:

$$\frac{d}{dt} v(t) = -Tv(t) + T\frac{d}{dt} y(t), \tag{3}$$

where $T \triangleq diag(1/\tau_1, 1/\tau_2, \dots, 1/\tau_l)$ is a matrix, and $\tau_k \in \boldsymbol{R}^1$ $(k = 1, 2, \dots l)$ is a design parameter selected by designers. In this paper, by using the approximate derivative of (3), we consider the PID control law:

$$u(t) = K_p(t) y(t) + K_i(t) \int_0^t y(t)dt + K_d(t) v(t), \tag{4}$$

instead of the one of (2).

Now by introducing two complementary variables $z_1(t)$ and $z_2(t)$ which are defined as

$$\begin{gathered} z_1(t) \triangleq x(t),\\ z_2(t) \triangleq \int_0^t y(t)dt,\end{gathered} \tag{5}$$

the PID control law of (4) can be transformed into

$$u(t) = K_p(t)Cz_1(t) + K_i(t)z_2(t)dt + K_d(t)v(t). \tag{6}$$

Next, we consider augmented vectors $z_\omega(t) = (z_1^T(t), z_2^T(t), v^T(t))^T$ and $y_\omega(t) = (y^T(t), z_2^T(t), v^T(t))^T$, and then we have

$$\begin{gathered}\frac{d}{dt} z_\omega(t) = A_\omega z_\omega(t) + B_\omega u(t) + \Gamma_\omega \omega(t),\\ y_\omega(t) = C_\omega x(t).\end{gathered} \tag{7}$$

In (7), $A_{\boldsymbol{\omega}} \in \boldsymbol{R}^{(n+2l)\times(n+2l)}$, $B_{\boldsymbol{\omega}} \in \boldsymbol{R}^{(n+2l)\times m}$, $\Gamma_{\boldsymbol{\omega}} \in \boldsymbol{R}^{(n+2l)\times p}$, and $C_{\boldsymbol{\omega}} \in \boldsymbol{R}^{3l\times(n+2l)}$, are the matrices given as

$$A_\omega = \begin{pmatrix} A, & 0, & 0,\\ C, & 0, & 0,\\ TCA & 0, & -T\end{pmatrix}, B_{\boldsymbol{\omega}} = \begin{pmatrix} B\\ 0\\ TCB\end{pmatrix}, \Gamma_{\boldsymbol{\omega}} = \begin{pmatrix} \Gamma\\ 0\\ TC\Gamma\end{pmatrix}, C_{\boldsymbol{\omega}} = diag(C, \quad I_l, \quad I_l). \tag{8}$$

Then one can see that introducing the matrix $K(t) = \left(K_p(t),\ K_i(t),\ K_d(t)\right)$ and some algebraic manipulations give (6) as (9):

$$u(t) = K(t)C_\omega z_\omega(t), \tag{9}$$

and the closed-loop system is given as

$$\begin{gathered}\boldsymbol{\frac{d}{dt} z_\omega(t) = (A_\omega + B_\omega K(t) C_\omega) z_\omega(t) + \Gamma_\omega \omega(t),} \\ \boldsymbol{y_\omega(t) = C_\omega z_\omega(t).}\end{gathered} \tag{10}$$

Now based on the existing results (e.g., Nagai et al., (2018).]) we will give the definition of the PID control with guaranteed L2 gain performance $\gamma < 0$ for the augmented system of (7) and the control input of (9).

Definition 1. For the augmented system of (7), the control input of (9) is said to be a PID control with guaranteed L_2 gain performance $\gamma > 0$ if the closed-loop system of (10) is internally stable and L_2-norm of the closed-loop system transfer function from the disturbance input $w(t)$ to the controlled output $y_\omega(t)$ is less than or equal to a positive constant $\gamma > 0$.

From the above, the design problem of the PID control law of (9) in this paper is reduced to one automatic adjustment laws for PID parameters with guaranteed disturbance attenuation performance of the expanded system of (7).

AUTOMATIC ADJUSTMENT OF THE PID PARAMETERS

To derive the automatic adjustment law of the PID parameters of (9), we consider the following function $V(t)$:

$$V(t) = z_\omega^T(t) P z_\omega(t) + Tr\{(K(t) - F)Q^{-1}(K(t) - F)^T\}. \tag{11}$$

In (11), $P \in \boldsymbol{R}^{(n+2l)\times(n+2l)}$ is a positive-definite symmetric matrix, $F \in \boldsymbol{R}^{m\times 3l}$ is a virtual matrix that is determined later, and $Q \in \boldsymbol{R}^{3l\times 3l}$ is a positive-definite weighting matrix selected by designers. Note that the virtual matrix F is introduced to derive the automatic adjustment law of the PID parameter and it is not directly reflected in the automatic adjustment law of PID parameters. The time-derivative of $V(t)$ along the trajectory of the closed-loop system of (11) are given as

$$\begin{aligned}\frac{d}{dt}V(t) = {} & z_\omega^T(t)\{P(A_\omega + B_\omega F C_\omega) + (A_\omega + B_\omega F C_\omega)^T P\} z_\omega(t) \\ & + 2 z_\omega^T(t) P B_\omega (k(t) - F) C_\omega z_\omega(t) + 2 z_\omega^T(t) P \Gamma_\omega \omega(t) \\ & + Tr\left\{\left(\frac{d}{dt}K(t)\right) Q^{-1} (K(t) - F)^T\right\} + Tr\left\{(K(t) - F) Q^{-1} \left(\frac{d}{dt}K(t)\right)^T\right\}.\end{aligned} \tag{12}$$

Let us consider the second term for the right-hand side of (12). If there exists a matrix Λ satisfying the relation

$$B_\omega^T P = \Lambda C_\omega, \tag{13}$$

then the time-derivative of $V(t)$ of (12) can be rewritten as

$$\begin{aligned}\frac{d}{dt}V(t) = {} & z_\omega^T(t)\{P(A_\omega + B_\omega F C_\omega) + (A_\omega + B_\omega F C_\omega)^T P\} z_\omega(t) \\ & + 2 z_\omega^T(t) C_\omega^T \Lambda^T (k(t) - F) C_\omega z_\omega(t) + 2 z_\omega^T(t) P \Gamma_\omega \omega(t) \\ & + Tr\left\{\left(\frac{d}{dt}K(t)\right) Q^{-1} (K(t) - F)^T\right\} + Tr\left\{(K(t) - F) Q^{-1} \left(\frac{d}{dt}K(t)\right)^T\right\}.\end{aligned} \tag{14}$$

Therefore, if we select the adjustment law of $K(t) \in \boldsymbol{R}^{m\times 3l}$

$$\left(\frac{d}{dt}K(t)\right)^T = -QC_\omega z_\omega(t)z_\omega^T(t)C_\omega^T\Lambda^T, \tag{15}$$

then the time-derivative of $V(t)$ of (14) can be rewritten as

$$\frac{d}{dt}V(t) = z_\omega^T(t)\{P(A_\omega + B_\omega F C_\omega) + (A_\omega + B_\omega F C_\omega)^T P\}z_\omega(t) + 2z_\omega^T(t)P\Gamma_\omega\omega(t). \tag{16}$$

Next, we introduce the following Hamiltonian so as to consider the disturbance attenuation performance:

$$H(t) = \frac{d}{dt}V(t) + y_\omega^T(t)y_\omega(t) - \gamma^2\omega^t(t)\omega(t). \tag{17}$$

In (17), if the Hamiltonian satisfies the condition $H(t) < 0$, then it means that the relation $\|y_\omega(t)\|^2 < \gamma^2\|\omega(t)\|^2$ holds, i.e., the disturbance attenuation performance $\gamma > 0$ is guaranteed. Thus, we consider the relation $H(t) < 0$. One can see from (14) that the condition $H(t) < 0$ can be rewritten as

$$\begin{pmatrix} z_\omega(t) \\ \omega(t) \end{pmatrix}^T \begin{pmatrix} P(A_\omega + B_\omega F C_\omega) + (A_\omega + B_\omega F C_\omega)^T P + C_\omega^T C_\omega & P\Gamma_\omega \\ P\Gamma_\omega & -\gamma^2 I_p \end{pmatrix} \begin{pmatrix} z_\omega(t) \\ \omega(t) \end{pmatrix} < 0. \tag{18}$$

Therefore, the disturbance attenuation performance γ is guaranteed provided that the following condition is ensured:

$$\begin{pmatrix} P(A_\omega + B_\omega F C_\omega) + (A_\omega + B_\omega F C_\omega)^T P + C_\omega^T C_\omega & P\Gamma_\omega \\ P\Gamma_\omega & -\gamma^2 I_p \end{pmatrix} < 0. \tag{19}$$

Moreover, by replacing $\gamma^* \triangleq \gamma^2$, the inequality of (19) is reduced to

$$\begin{pmatrix} \boldsymbol{P(A_\omega + B_\omega F C_\omega) + (A_\omega + B_\omega F C_\omega)^T P + C_\omega^T C_\omega} & \boldsymbol{P\Gamma_\omega} \\ \boldsymbol{P\Gamma_\omega} & \boldsymbol{-\gamma^* I_p} \end{pmatrix} < \boldsymbol{0}. \tag{20}$$

As a result, we obtain the following theorem for the proposed PID control system:

Theorem 1. Consider the augmented system of (7). If there exist a positive definite symmetric matrix $P \in \boldsymbol{R}^{(n+2l)\times(n+2l)}$, a virtual matrix $F \in \boldsymbol{R}^{m\times 3l}$ and a positive constant γ^* which satisfy the matrix inequality (20), the adjustment law of PID parameter is determined as (15) and the internal stability of the closed-loop system and disturbance attenuation performance of $\gamma \left(= \sqrt{\gamma^*}\right)$ is guaranteed.

Remark 1. The inequality condition of (20) isn't an LMI. In this paper, we adopt the design method of static output feedback controller presented in Benton et al., (1999). Namely, by using the design procedure shown in it, a positive definite symmetric matrix $P \in \boldsymbol{R}^{(n+2l)\times(n+2l)}$, a virtual matrix $F \in \boldsymbol{R}^{m\times 3l}$ and a positive constant γ^* which satisfy the matrix inequality (20) are derived.

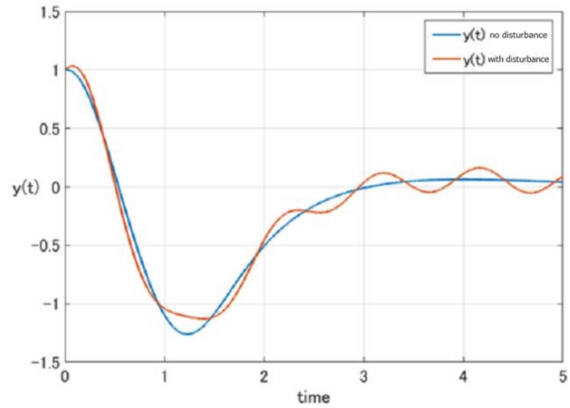

Figure 1 : time response of the output

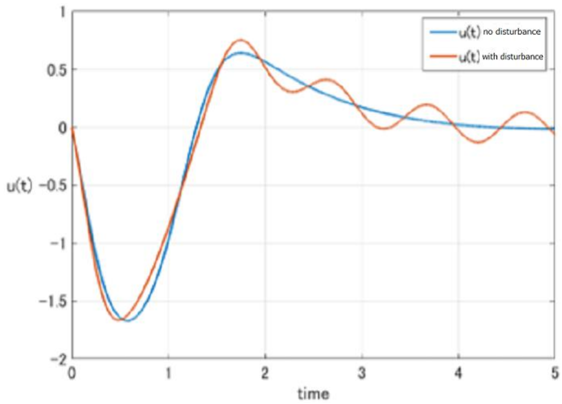

Figure 2 : time response of the input

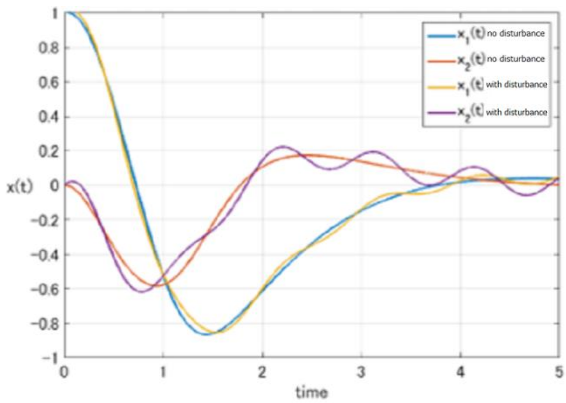

Figure 3 : time response of the state

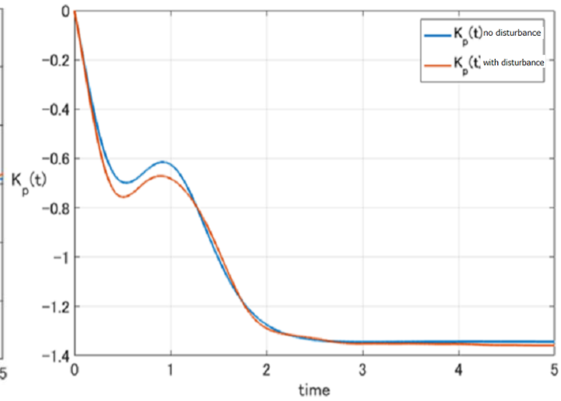

Figure 4 : time response of the $K_p(t)$

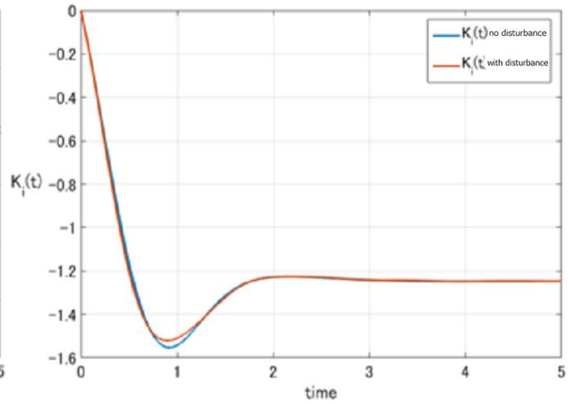

Figure 5 : time response of the $K_i(t)$

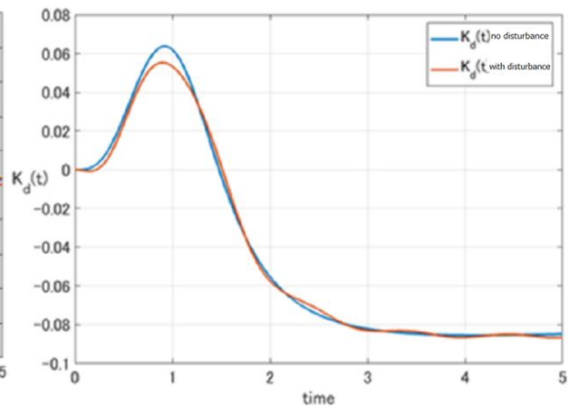

Figure 6 : the time response of the $K_d(t)$

NUMERICAL SIMULATION

In order to demonstrate the proposed control system, a simple numerical simulation is run. In this numerical example, we consider a MIMO system described as

$$\frac{d}{dt}x(t) = \begin{pmatrix} 0.1 & 1.0 \\ 0.0 & 2.0 \end{pmatrix} x(t) + \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix} u(t) + \begin{pmatrix} 0.10 \\ 0.25 \end{pmatrix} \omega(\mathrm{t}).$$

$$y(t) = (1.0 \quad 1.0)x(t),$$

Moreover, we set the parameter τ_1 as 10 for the approximate derivative of (3), and the design parameter Q is selected as $Q = I_3$. Additionally, initial values are set as $v(0) = 0, K_p(0) = K_i(0) = K_d(0) = 0$ and $x(0) =$

$(x_1(0),\ x_2(0))^T = (1, 0)^T$, respectively. Then, we simulate two patterns; (1) the disturbance is given by $\omega(t) = 2\cos(2\pi t)e^{-0.001t}$ and (2) no disturbances.

By using the Benton et al., 1999, the following matrices and a scalar γ^* which satisfy the conditions of (13), and (20) are derived,

$$F = (2556.823 \quad -2153.270 \quad -9.624), \qquad \Lambda = (2.655 \quad 2.236 \quad 0.010),$$
$$P = \begin{pmatrix} 2.701 & 0.453 & 0.779 \\ 0.453 & 2.699 & 1.501 \\ 0.779 & 1.501 & 5.861 \end{pmatrix}, \qquad \gamma^* = 4.279(\gamma = 2.069).$$

The figures 1-6 are the time-histories of the numerical simulation. We find that he PID parameter $K_p(t), K_i(t)$ and $K_d(t)$ are adjusted automatically, and the closed-loop system is asymptotically stable.

CONCLUSION

In this paper, we have shown a new automatic adjustment law of the PID parameters with disturbance attenuation performance for Multiple-Input Multiple-Output linear systems. The sufficient conditions for the existence of the PID control system which has the disturbance attenuation performance can be reduced to the condition of the matrix inequality. Furthermore, the effectiveness of the proposed PID control system has been presented through a simple numerical example.

The future research subjects are establishing much appropriate derivation method of the matrix Λ, and extension of the proposed design approach for PID control systems to I-PD control strategy.

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